Appendix E

Methods Used for AECOM Present Worth Calculations

METHODS USED FOR AECOM PRESENT WORTH COST CALCULATIONS

Costs have been estimated using two different methods including the Simple Present Worth (SPW) Method and the Life-Cycle Cost (LCC) Method. Differences in the two methods include the following:

- The SPW Method is based upon a 20-year planning period, whereas the LCC Method is based upon a 50-year planning period; and
- The LCC Method accounts for an inflation rate of 3% whereas the SPW method does not account for inflation.

Assumptions common to both methods include:

- Non-construction capital costs are assumed to equal 30% of construction capital costs; and
- An interest rate of 7.375% is assumed.

The equations used for each method for estimating capital costs and annually recurring costs, respectively, are presented below.

SPW METHOD

Capital Costs

Condition 1: The entire service life of the improvement falls within the 20-year planning period (Service life ≤ 20 – [year "X" of planning period in which capital cost is paid]).

Equation:

$$PW_{20} = F/(1+I)^X \qquad (1)$$

where:

PW_{20} = 20-year present worth of capital cost
F = Future worth of capital cost
I = Interest rate = 7.375%
X = Year of the planning period in which capital cost is paid (i.e., 2,3,4, …)

Equation Explanation:

If one hundred dollars are invested at an interest rate of 7.375% for one year, the investment would yield ($100 x 1.07375 = $107.38). After another year, the $107.38 invested at the same interest rate would yield ($107.38 x 1.07375 = $115.30). This example shows compounding annual interest, in which interest accrues annually on both the original investment and on any

interest earned during previous years. The original investment is multiplied by 1.07375 for each year that it is invested. The equation for this scenario may therefore be written as:

$$F = P(1.07375)^X \quad (2)$$

Rearranging the equation yields:

$$P = F/(1.07375)^X \quad (3)$$

This is the same as equation (1). Thus, the calculated present worth is the amount that would be have to be invested now to yield the specified future worth in year X assuming an interest rate of 7.375%. The reader should note that for purposes of this project, for year 1, the value of X is zero. For all subsequent years, the value of X is the number of the year (i.e., 2,3,4, …). The reason for this difference is that agencies typically equate the amount of money available at the start of a project to that in year 1. In actuality, the money at the start of the project accrues interest over the first year.

Condition 2: Service life of the improvement extends beyond the 20-year planning period (Service life > 20 – [year "X" of planning period in which capital cost is paid]).

<u>Equation</u>:

$$PW_{20} = F([1/\{(1+I)^X\}]-[\{(S+X-20)/S\}/\{1+I\}^{20}]) \quad (4)$$

where:

PW_{20} = 20-year present worth of capital cost
F = Future worth of capital cost
I = Interest rate = 7.375%
X = Year of the planning period in which capital cost is paid (2,3,4, …)
S = Number of years of service life after capital cost is paid

<u>Equation Explanation</u>:

Equation (4) may be rewritten as follows:

$$PW_{20} = (F\ [1/\{(1+I)^X\}])-(F[\{(S+X-20)/S\}/\{1+I\}^{20}]) \quad (5)$$

The first term is equivalent to equation (1) and yields the present worth of a future capital cost expended in year X assuming an interest rate I.

The second term can be broken into three parts:

$$F[\{(S+X+20)/20\}/\{1+I\}^{20}] = F * ([S+X-20]/S) * (1/[1+I]^{20}) \quad (6)$$

In this equation, the second term represents the ratio of the number of years that the service life extends beyond the planning period to the service life. This ratio reflects the proportion of the

service life that is available at the end of the 20-year planning period. Hence the future worth multiplied by this term yields the salvage value or the amount which would be obtained by selling the item after the twentieth year of the project. The third term here serves to back-calculate this salvage value to present worth assuming an interest rate of 7.375%. Therefore, equation (5) represents cost and equation (6) represents the salvage value that must be subtracted from it. Again, the reader should note that for year 1, the value of X is zero. For all subsequent years, the value of X is the number of the year when the project begins (i.e., 2,3,4, …). The present worth of the salvage value is subtracted from the present worth of the cost of the project since salvage value represents money returned and not a cost.

Annually Recurring Costs

Equation:

$$PW_{20} = \sum A/(1+I)^X \qquad (7)$$

where:

PW_{20} = Summation of present worth of annually recurring costs from each year
A = Amount of annually recurring cost
I = Interest rate = 7.375%
X = Year of planning period in which annually recurring costs begin

Equation Explanation:

As explained for equation (1), if one starts with $100 and invests it for X number of years, the final value of the money after X years with an interest rate of I is given by:

$$F = P(1+I)^X \qquad (8)$$

Rearranging this equation yields:

$$P = F/(1+I)^X \qquad (9)$$

which is similar to equation (7). One can think of the annual recurring payment A as a series of separate payments made for a number of years in the future. Although each payment is the same amount in 1998 dollars, payment in each year has to be back-calculated from a different year to its present worth therefore resulting in a different present worth value for each payment. The summation of the separate present worth values for each of the years is the total present worth of the annually recurring costs.

LCC METHOD

Capital Costs

Condition 1: : The entire service life of the improvement falls within the 50-year planning period (Service life ≤ 50 – [year "X" of planning period in which capital cost is first paid).

Equation:

$$PW_{50} = F([1+f]^{X-1}/[1+I]^{X}) \quad (10)$$

where:

PW_{50} = 50-year present worth of capital cost
F = Future worth of capital cost
f = Inflation rate = 3%
X = Year of the planning period in which capital cost is paid (i.e., 2,3,4, …)
I = Interest rate = 7.375%

Explanation of Equation:

Here, as with equation (1), the present worth is back-calculated from the future worth by dividing the future worth by the interest rate compounded for X number of years. In this case, the term:

$$[1+f]^{X-1}$$

accounts for inflation. The future worth, which occurs in year X, is first inflated to account for X-1 years of inflation. The capital cost at the beginning of the project is assumed to be valid for the entire first year of the planning period. Therefore inflation of capital cost does not begin until the *second* year. As such, the period of inflation is X-1 years. This inflated value is then back-calculated to present worth.

Condition 2: : Service life of the improvement extends beyond the 50-year planning period (Service life > 50 – [year "X" of planning period in which capital cost is first paid).

Equation:

$$PW_{50} = F * ([1+f]^{[X-1]}) * ([I*\{(1+I)^{S}\}]/[\{(1+I)^{S}\}-1]) * ([\{(1+I)^{(50-X)}\}-1]/ [I*\{(1+I)^{(50-X)}\}]) * (1/[\{1+I\}^{X}]) \quad (11)$$

where:

PW_{50} = 50-year present worth of capital cost
F = Future worth of capital cost
f = Inflation rate = 3%
X = Year of the planning period in which capital cost is paid (i.e., 2,3,4, …)

I = Interest rate = 7.375%
S = Number of years of service life after capital cost is paid

This equation may be better understood by examining each of the 5 terms:

- F: This term represents the capital cost expended in a future year X (i.e., 2, 3, 4, …).
- $([1+f]^{[X-1]})$: This term accounts for inflation of the current value of the money to the year X.
- $([I*\{(1+I)^S\}]/[\{(1+I)^S -1])$: This term multiplied with the first two terms described above calculates the annual cost that would have to be expended over the service life of the project to equal the inflated future capital cost.
- $([\{(1+I)^{(50-X)}\}-1]/ [I*\{(1+I)^{(50-X)}\}])$: This term multiplied with the three terms above back-calculates the inflated future capital cost from the calculated annual cost but only for annual cost expended for the portion of the service life of the project within the 50 year planning period.
- $(1/[\{1+I\}^X])$: This term multiplied with all the other terms allows present worth to be back-calculated from the inflated future capital cost given an interest rate I.

Both equations (10) and (11) are used starting with the second year. For year 1, the capital cost is specifically left unadjusted. Again, the reason for this difference is that agencies typically equate the amount of money available at the start of a project to that in year 1. In actuality, the money at the start of the project will accrue interest over the first year.

<u>Annually Recurring Costs</u>

<u>Equation</u>:

$$PW_{50} = \sum A([1+f]^{X-1}/[1+I]^X) \qquad (12)$$

where:

PW_{50} = Summation of present worth of annually recurring costs from each year
A = Amount of annually recurring cost
f = Inflation rate = 3%

I = Interest rate = 7.375%
X = Year of planning period in which annually recurring costs begin

<u>Equation Explanation</u>:

Here again, the equation resembles that associated with equations (1) and (3):

$P = F/(1+I)^X$

The additional term:

$[1+f]^{X-1}$

accounts for inflation as explained for equation (10). The annual payment for each year X must be inflated for X-1 years. The inflated value is then back-calculated to present worth. Each payment is back-calculated separately. The sum of these present worth values yield the total present worth of the recurring annual cost.